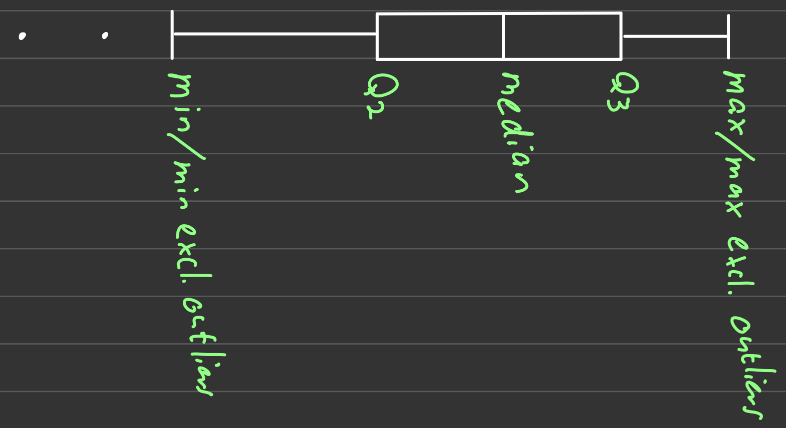
5-point summary

- min
- Q_1
- Median
- Q_3
- max

Boxplot



Removing outliers

- if outside $[q_1 - 1.5(q_3 - q_1),\ q_3 + 1.5(q_3 - q_1)]$
  i.e. more than $1.5 * IQR$ from q_1 / q_3

Probability dist.

Discrete

i know

Continuous

Cumulitive dist of X

- $F_X(a) = \int_{-\infty}^{a} f_X(k)\,dk = P(X \leq a)$
- $F_X(\infty) = 1$

Quantiles

Smallest ϕ_p such that

for discrete

$$\phi_p : \int_{-\infty}^{\phi_p} f_X(a)\,da \geq p$$

$$P(X \leq \phi_p) = F_X(\phi_p) \geq p$$

Transform to std normal

$Y \sim n(\mu, \sigma^2)$

$Z \sim n(0, 1)$

$$Z = \frac{Y - \mu}{\sigma}$$

Expected value

$$E(X) = \int_{-\infty}^{\infty} x \times f_X(x)\, dx$$

constant

$$E(c) = \int_{-\infty}^{\infty} c\, f_X(x)\, dx = c \int_{-\infty}^{\infty} f_X(x)\, dx = c \times 1$$

Uniform distribution

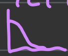
$Y \sim unif(a, b)$

$E(Y) =$

$$var(Y) = \frac{1}{12}(b-a)^2$$

Gamma distribution

shape, moves bump left or right
$\alpha = 1$, bump



Scale

$Y \sim gamma(\alpha, \beta)$

$$f_Y(y) = \frac{y^{\alpha-1} e^{-\frac{y}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}, \qquad y > 0$$

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y}\, dy$$

$\Gamma(1) = 1$

if $\alpha \in \mathbb{N}$, $\quad \Gamma(\alpha) = (\alpha - 1)!$

$E[Y] = \alpha\beta$

$var(Y) = \alpha\beta^2$

$$E(Y) = \int_{-\infty}^{\infty} y f_Y(y)\, dy = \int_0^{\infty} \frac{y^{a} e^{-y/\beta}}{\beta^{\alpha}\Gamma(\alpha)}\, dy$$

$$= \frac{\beta^{\alpha+1}\Gamma(\alpha+1)}{\beta^{\alpha}\Gamma(\alpha)} \int_0^{\infty} \frac{y^{\alpha} e^{-y/\beta}}{\beta^{\alpha+1}\Gamma(\alpha+1)}\, dy$$

let $\alpha + 1 = k$

$a = k-1$

$$= \frac{\beta^{\alpha+1}\Gamma(\alpha+1)}{\beta^{\alpha}\Gamma(\alpha)} \int_0^{\infty} \frac{y^{k-1} e^{-y/\beta}}{\beta^{k-1}\Gamma(k)}\, dy$$

integrating full range of gamma$(k-1, \beta)$ $\rightarrow 1$

$$= \frac{\beta^{a+1}\Gamma(\alpha+1)}{\beta^{\alpha}\Gamma(\alpha)} =$$

$Y \sim$ gamma(α, β)

$$\text{var}(Y) = E[Y - E[Y]]^2 = E[Y^2] - E[Y]^2$$

$$E[Y^2] = \int_{-\infty}^{\infty} y^2 f_Y(y)\, dy$$

$$= \int_{-\infty}^{\infty} \frac{y^{\alpha+2} e^{-y/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$$

$$= \frac{\beta^{\alpha+2}\Gamma(\alpha+2)}{\beta^{\alpha}\Gamma(\alpha)} \int_{-\infty}^{\infty} \frac{y^{a+2} e^{-y/\beta}}{\beta^{a+2}\Gamma(a+2)}\, dy$$

let $k = a + 2$

$$= \frac{\beta^{a+2}\Gamma(\alpha+2)}{\beta^{a}\Gamma(\alpha)} \int_{-\infty}^{\infty} \underbrace{\frac{y^{k} e^{-y/\beta}}{\beta^{k}\Gamma(k)}\, dy}_{=1}$$

Chi squared

$$\chi^2_v = \frac{2\,\text{gamma}(\frac{v}{2}, \overset{=2}{\beta})}{\beta}$$

$$\chi^2_8 = \frac{2\,\text{gamma}(4,4)}{4}$$

$$2\chi^2_8$$

- interpolate from table

$$\begin{array}{ccc} 7 & & 1 \\ 0.5 & \uparrow & 0.75 \\ & 6 & \end{array}$$

$$0.5 + \frac{0.75 - 0.5}{-6} \times (6-7)$$